

Sounding Rocket Boost-Phase Gust Angle of Attack

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This paper derives approximate angle-of-attack statistics suitable for boost-phase structural loading estimates on unguided, fin-stabilized sounding rockets. Sounding rockets are assumed to lift off with a large constant vertical acceleration. Their rigid-body rotations are modeled as undamped short-period motions without any velocity vector rotation; the only source of pitch/yaw torques is via aerodynamic static stability. The vertical acceleration causes the dynamic pressure to increase rapidly, leading to time-varying coefficients in the short-period equations and, therefore, gust responses that require nonstationary analyses. Transforming the independent variable from time to altitude enables calculation of a simple lateral velocity sinusoidal gust impulse response function. Next, the total response for a single instantiation is found by superposition of all its gust impulses. Then, convolution to find the variance in transverse velocity is found based on the Dryden gust autocorrelations. A closed-form result for the standard deviation in boost-phase gust angle of attack is obtained and compared with both its high-altitude asymptote and the classical sharp-edged (step function) gust response. At altitudes above about two pitch wavelengths, the asymptote provides an accurate result, whereas the classical sharp-edged gust model significantly underestimates the gust response, except for regions very near the ground.

Nomenclature

a	= axial acceleration, m/s^2
C_{Mq}	= pitch moment coefficient derivative with respect to $(\omega_y d/2U)$, rad^{-1}
$C_{M\alpha}$	= pitch moment coefficient derivative with respect to angle of attack, rad^{-1}
C_{Nr}	= yaw moment coefficient derivative with respect to $(\omega_z d/2U)$, rad^{-1}
$C_{N\beta}$	= yaw moment coefficient derivative with respect to angle of sideslip, rad^{-1}
$C_{Y\beta}$	= Y force coefficient derivative with respect to angle of sideslip, rad^{-1}
$C_{Z\alpha}$	= Z force coefficient derivative with respect to angle of attack, rad^{-1}
d	= aerodynamic reference length; body diameter, m
F_z	= z component of external force
h	= altitude, m
I_P	= moment of inertia about the pitch or yaw axis, kg-m^2
L	= launcher length, m
l_G	= longitudinal turbulence scale length, m
l_G^*	= pseudotransverse turbulence scale length, m
M_y	= y component of external moment
m	= rocket mass, kg
q	= dynamic pressure, kg/m^2
R	= autocorrelation function, m^2/s^2
S	= aerodynamic reference area, m^2
t	= time from liftoff, s
U	= axial (x axis) velocity, m/s
V_G	= wind (gust) velocity in the z direction, m/s
V_z	= inertial velocity in the z direction, m/s
$\text{var}()$	= variance of (); mean square of (); $\langle((())^2)\rangle$.
x, y, z	= body-fixed axes, x (roll) along the axis of symmetry, y and z (pitch & yaw) forming an orthogonal triad
z	= altitude dummy variable, m
α	= angle of attack, rad

λ_G	= transverse turbulence (gust) reciprocal scale length, m^{-1}
λ_P	= pitch/yaw wave number, rad/m
ρ	= atmospheric mass density, kg/m^3
σ_G	= standard deviation in wind (gust) velocity, m/s
σ_α	= planar standard deviation in angle of attack, rad
ω_Y	= angular rates about the pitch axis, rad/sec
$()_o$	= value of () at the time of separation from the launcher
$\langle \rangle$	= ensemble average of ()

Note that the right-hand rule is the sign convention used for moments and angular rates.

Introduction

THE main motivation behind this analysis is the estimation of sounding rocket boost-phase structural loads. Sounding rocket structural loads are driven by two kinds of perturbation: those due to the various structural misalignments that are best considered in a body-fixed frame and those due to atmospheric motions that are most easily analyzed in an Earth-fixed frame. Atmospheric motions, in turn, have two very different varieties: synoptic scale winds and gusts [1,2]. The atmospheric motions due to weather are called winds. For the most part, winds tend to blow in a nearly horizontal direction. They are characterized by correlation over large distance scales, often thousands of kilometers, and slow temporal rates of change over several days. The other variety of motion, gusts, arises from turbulent mixing within the atmosphere. When remote from the ground, gusts are three-dimensional and commonly considered isotropic. Their correlation distance scale is only hundreds of meters, and their temporal correlation scale is tens of seconds.

Sounding rockets initially encounter synoptic scale (weather) winds at the bottom of the planetary boundary layer where their velocities are small [1]. Therefore, winds can be safely neglected in estimating boost phase structural loads. On the other hand, based on experience [3], atmospheric gusts are the dominant Earth-fixed perturbations influencing sounding rocket structural loads. Turbulence (mixing) can be especially intense low in the planetary boundary layer, thus leading to significant structural loading in the early phases of rocket flight.

This analysis is in three parts. First, the response to an impulsive gust is found by integrating the approximate equations of motion. These are linear with time-varying coefficients due to the rapid increase of dynamic pressure immediately after launch. Changing the independent variable from time to altitude generates equations with constant coefficients. The response to an impulsive gust is then

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estimated. Second, gust statistics are described as Gaussian, frozen, isotropic, and stationary with an approximate Dryden autocorrelation function [1,4]. The Dryden autocorrelation function is used because, although it is empirical, it is simple in form, and it captures gust behavior reasonably well. The assumption of a frozen turbulence field is easy to justify because rockets accelerate through the atmosphere much faster than gusts evolve. Third, the impulse response and autocorrelation function are convolved to estimate the gust angle-of-attack standard deviation.

This is a simplified analysis. The ultimate rationale for its adoption is that the angle-of-attack response depends on gust (not wind) velocity statistics, which are known to be functions of location, season, and atmospheric synoptic state [1]. Because there is little or no long-term a priori launch day knowledge of some of these variables, there is no great value in using highly accurate models if the result is necessarily corrupted by inaccurate environmental knowledge. Classically, gust responses have been modeled using a sharp-edged step function or a 1 - cosine function. But, no such deterministic gusts are found in nature. It follows that responses estimated from such gust models should be viewed with very low confidence. The purpose of this paper, then, is to develop a simple, easy-to-use, physics-based estimate of sounding rocket boost-phase gust response statistics.

Impulsive Gust Response

The coordinate frame used in this analysis, shown in Fig. 1, is Earth-fixed in roll but allowed to pitch and yaw with the rocket body. Its origin follows the rocket center of mass as it accelerates under the influence of thrust. The idea is to keep the Earth-fixed perturbing effects, synoptic winds, and gusts in an invariant plane in the coordinate system.

The key assumptions are as follows. First, the rocket configuration can be characterized as slender with pitch-yaw symmetry. The rocket's roll moment of inertia is negligibly small compared with the pitch/yaw moment of inertia because its fineness ratio is 12 to 20, and the pitch moment of inertia over the roll moment of inertia \approx fineness ratio squared. Next, ω_y and α are both small compared with unity; sounding rocket short-period motions are dynamically very linear with a few stark, shocking exceptions, such as roll lock-in. The only significant aerodynamic pitch/yaw moments and forces are those due to static stability ($C_{M\alpha}$ and $C_{N\beta}$). According to Etkin [5], if the roll moment of inertia is negligibly small compared with the pitch/yaw moments of inertia, the dynamic equations of motion then decouple into a pitch set and a yaw set, thus greatly simplifying the analysis. The component of gravity along the y and z axes may be neglected because we have implicitly restricted our attention to short-period motion, and gravity, apart from its affect on axial acceleration, can be neglected as it primarily influences the trajectory itself (phugoid mode).

Also, the rocket's short-period damping, although positive, almost vanishes; that is, terms in C_{Mq} , $C_{Z\alpha}$, C_{Nr} , $C_{Y\beta}$, and jet damping may be neglected. Direct numerical analysis of several typical sounding rocket configurations [3] shows that their short-period damping is usually less than 1% of critical. The neglected damping terms are not literally zero; they are merely very tiny. The small short-period damping ensures that the effects of a gust can be observed ringing

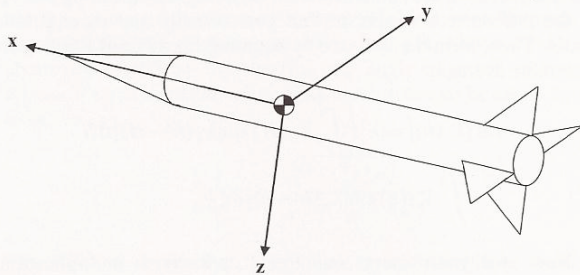


Fig. 1 Coordinate frame.

long after the original encounter. A simple calculation shows that, for damping equal to about 1% of critical, it will take almost two pitch wavelengths for a transient gust response to lose 10% of its amplitude. In other words, because the actual short-period damping is usually less than 1% of critical, as long as we restrict ourselves to altitudes near the launcher, neglecting short-period damping ought to be a fairly good approximation.

A large, constant axial acceleration implies the equations have profoundly time-varying coefficients. Changes in rocket mass, air density, center-of-gravity shifts, and Mach-number-driven variations in stability derivatives are neglected, leaving the change in dynamic pressure as the dominant cause of time-varying coefficients.

Etkin [5] provides the source for the rocket's equations of motion, starting with the assumed solution for constant acceleration:

$$U = at = \sqrt{2ah} \quad (1)$$

Then, the approximate short-period pitching and plunging equations, written for nonrolling axes otherwise fixed to a rigid body, can be readily found [5] neglecting pitching and plunging damping:

$$M_y = I_P d\omega_y/dt = qSdC_{M\alpha} \left(\alpha - \frac{V_G}{U} \right), \quad \text{and} \quad (2)$$

$$F_Z/m = dV_Z/dt - U\omega_y = 0$$

Using the customary definitions of the angle of attack, $\alpha \equiv V_Z/U$, and of the dynamic pressure, $q = \frac{1}{2}\rho U^2$, it is found that

$$I_P d\omega_y/dt = \frac{1}{2}\rho SdC_{M\alpha} U(V_Z - V_G), \quad \text{and} \quad dV_Z/dt - U\omega_y = 0 \quad (3)$$

An important simplification results from changing the independent variable from time to distance along the flight path, which, if vertical flight is assumed, amounts to altitude. Using the chain rule to obtain linear equations of motion with constant coefficients per Hoult [6],

$$d/dt = dh/dt * d/dh, \quad dh/dt = U d/dh$$

Then,

$$I_P d\omega_y/dh = \frac{1}{2}\rho SdC_{M\alpha}(V_Z - V_G), \quad \text{and} \quad dV_Z/dh - \omega_y = 0$$

Defining the pitch wave number λ_p in radians per meter,

$$\lambda_p^2 = -\frac{1}{2}\rho SdC_{M\alpha}/I_P \quad (4)$$

we have

$$d\omega_y/dh = -\lambda_p^2(V_Z - V_G), \quad \text{and} \quad dV_Z/dh - \omega_y = 0 \quad (5)$$

Eliminating ω_y from this pair, we find

$$d^2V_Z/dh^2 + \lambda_p^2 V_Z = \lambda_p^2 V_G \quad (6)$$

Next, solutions to Eq. (6) are needed for an arbitrary gust profile $V_G(h)$. Now, suppose $V_G(h)$ vanishes everywhere except for in an infinitesimal slice of altitude starting at altitude η and ending slightly higher at altitude $\eta + dh$. Suppose the gust impulse amplitude in this altitude region is V_G . Then, the impulsive gust solution to Eq. (6), valid for altitudes h above η is

$$V_Z = \lambda_p V_G(\eta) \sin[\lambda_p(h - \eta)] \quad (7)$$

Decompose any arbitrary gust profile $V_G(h)$ into a set of layered impulse functions. Then, because Eq. (6) is linear, the total $V_Z(h)$ is just a superposition of all the gust impulses acting at altitudes below h :

$$V_Z(h) = \lambda_p \int_L^h V_G(\eta) \sin[\lambda_p(h - \eta)] d\eta \quad (8)$$

Gust Phenomenology

In general, a power spectral density (PSD) plot of the atmospheric wind field [2] will show two major peaks associated with processes for turbulence (gusts) and weather (synoptic scale winds). Synoptic scale winds are constrained by gravity to lie in the horizontal plane, whereas gusts tend to be fully three-dimensional and isotropic when not too close to the ground. Typical distance scales are 4000 km for weather and 600 m outside the planetary boundary layer for gusts. Four temporal orders of magnitude also separate these processes. Although both are of interest to a rocket engineer, gusts usually contribute significantly more to the angle-of-attack response.

Now, consider the issue of stationarity. For many aerospace applications (aircraft and large rockets), it is reasonable to assume stationary statistical processes and to exploit the benefits of frequency domain analyses. Sounding rockets, however, are a breed apart. Their large axial acceleration implies large changes in flight condition while ringing from earlier gusts is still happening. Therefore, a nonstationary analysis is required. That is, we must work with autocorrelation functions rather than PSD functions to describe gust statistics.

During World War II, Dryden [4] showed empirically that the longitudinal autocorrelation function for wind-tunnel turbulence could be accurately described by a simple exponential function of the separation distance together with Gaussian statistics. It should be noted that the Dryden PSD has an asymptotic log-log slope of -2 . Later, the Kolmogorov model [5] showed, by applying dimensional analysis to turbulence cascades, that the asymptotic log-log slope should be $-5/3$. Much of the available empirical data would fit either model equally well. The clincher is that the inverse Fourier transform of the Kolmogorov PSD (to find the autocorrelation function) yields difficult-to-use Bessel functions [5]. The Dryden exponential autocorrelation is simple and easy-to-use and has, therefore, been chosen for this analysis. Even though Dryden's original research is almost 70 years old, it is still useful for gust response analyses [1,5]. However, for the current application [1,5], the transverse autocorrelation function (relation between two gust velocities normal to the unperturbed velocity vector) is what's needed. Fortunately, Batchelor [7] provided a simple relationship between the two, which is valid for incompressible flow.

The turbulent gust model used here assumes that gusts can be described with each velocity component having a one-dimensional Gaussian probability distribution with zero mean. The three orthogonal velocity components are all statistically independent. We will assume that the turbulence is isotropic; that is, its properties are the same in all directions, even though this is not strictly true within the planetary boundary layer. Turbulence is assumed to be homogeneous; that is, its statistics are the same everywhere, and it is stationary with no temporal variation in its properties.

When analyzing vehicles flying through a turbulence field at high speeds, turbulence can be modeled as frozen; that is, its properties do not change significantly while the rocket flies from one altitude to a higher one. Then, temporal correlations can be neglected, leaving only spatial ones.

In a longitudinal gust autocorrelation function, the two velocities are separated and collinear, whereas, for a transverse gust autocorrelation function, the two velocities are parallel but offset from each other. The Dryden longitudinal gust autocorrelation function [1,4,5] is

$$R = \sigma_G^2 \exp(-\Delta x / l_G) \quad (9)$$

and the corresponding transverse gust autocorrelation function, obtained using Batchelor's theorem [7], is

$$R = \sigma_G^2 \exp(-\Delta z / l_G) \left(1 - \frac{1}{2} \Delta z / l_G\right) \quad (10)$$

Here, Δx and Δz are the absolute values of the longitudinal and transverse separation distances between the two points whose velocities are correlated. Because rockets fly nearly vertically, only the two horizontal vector components cause significant aerodynamic

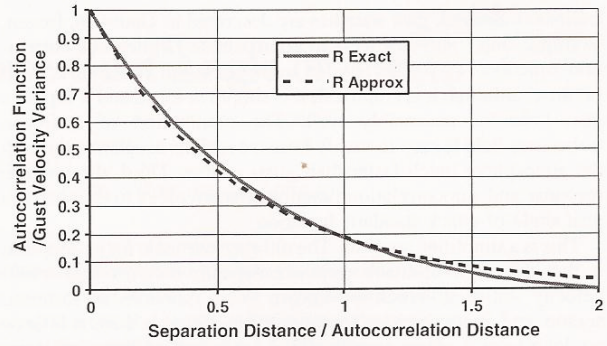


Fig. 2 Transverse autocorrelation function normalized by the gust velocity variance.

loads, and they are statistically independent [1,5], each with the same autocorrelation function given by Eq. (10).

As suggested in the literature [1], for the small arguments commonly encountered in small sounding rocket work, the transverse autocorrelation function looks a lot like the longitudinal function with $0.59x$ as the turbulence scale distance. In any case, because use of the Dryden function can only be justified by its close match to the experimental data, this further approximation will give acceptably accurate results. Then, the pseudoscale distance for transverse correlation is

$$l_G^* \cong 0.59 l_G \quad (11)$$

Figure 2 shows that this is a fairly close approximation. Here, "R Exact" is given by Eq. (10), whereas "R Approx" is given by Eq. (9) but with the modified correlation scale distance of Eq. (11).

The final issue is how to quantify the parameters λ_G and σ_G . The literature [1] provides a modern compilation of how these vary with their contextual situation. These also vary significantly through the planetary boundary layer as also described in the literature [8]. Without more specific insights, one could take typical values [1,8] above 200 m altitude in the planetary boundary layer as

$$\sigma_G = 1 \text{ m/s, and } l_G = 300 \text{ m}$$

Then, $\lambda_G = 300 \text{ m}^{-1} = 0.003333 \text{ m}^{-1}$ (longitudinal) and $\lambda_G^* \equiv 1/(0.59 * 300 \text{ m}) = 0.005650 \text{ m}^{-1}$ (transverse).

Gust Response Statistics

First, because gusts have zero mean velocity [1,5], i.e., $\langle V_G(h) \rangle = 0$, the mean gust angle of attack at any altitude will vanish. However, the variance does not vanish. This situation is analogous to a drunkard's random walk on a sidewalk. There is a 50% chance that each of his steps will be to the right (or left). After a large number of steps, he will, on average, not have travelled away from his starting point. But the variance (mean square) in the distance travelled is independent of right vs left, and it continues to increase without bound. Sounding rocket gust response statistics behave exactly the same way.

Using the impulse response function and the approximate Dryden transverse gust autocorrelation function, the gust lateral velocity variance is found in a straightforward way. Begin by recalling that V_G is the difference between the full gust velocity and its ensemble mean. Then, form the variance by squaring Eq. (8) and forming its ensemble average:

$$\begin{aligned} \text{var}[V_z(h)] = & \lambda_p^2 \left\langle \int_L^h V_G(H) \sin[\lambda_p(h-H)] dH \right. \\ & \left. * \int_L^h V_G(\eta) \sin[\lambda_p(h-\eta)] d\eta \right\rangle \end{aligned}$$

Now, we must carry out three operations: multiplication, integration (summing) over the altitude region, and ensemble

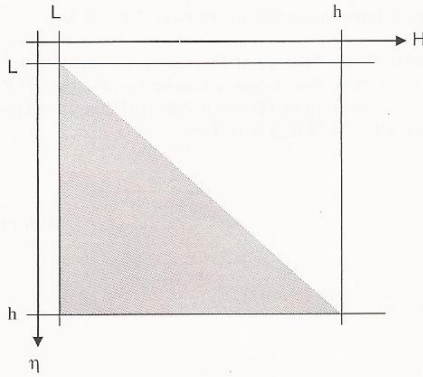


Fig. 3 Integration region.

averaging. By exercising care, the order in which these three operations are performed can be varied. Begin by expressing the integrals as the limit of sums and multiplying them together, term by term. Next, form the ensemble average of every product term in the double sum, noting that the Dryden correlation function depends on the absolute value of the separation distance. Before integration, examine Fig. 3 carefully. Every product term at (H, η) on one side of the 45 deg line is matched by another on the opposite side having the same value. Thus, the integral over the shaded region of Fig. 3 has the same value as that over the clear region, and it is only necessary to integrate over the shaded area in Fig. 3 and multiply by 2. In the shaded region, η will always be $\geq H$, thus removing the need to consider absolute values in integration.

Finally, pass back to the limit to obtain

$$\begin{aligned} \text{var}[V_z(h)] &= 2\lambda_p^2 \sigma_G^2 \int_L^h \sin[\lambda_p(h-H)] dH \\ &\quad * \int_H^h \exp[\lambda_G^*(H-\eta)] \sin[\lambda_p(h-\eta)] d\eta \end{aligned} \quad (12)$$

After integration, we obtain

$$\begin{aligned} \text{var}[V_z(h)] &= \frac{\lambda_p^2 \sigma_G^2}{\lambda_p^2 + \lambda_G^2} \left\{ \lambda_G^*(h-L) - \frac{\lambda_G^*}{2\lambda_p} \sin[2\lambda_p(h-L)] \right. \\ &\quad - \sin^2[\lambda_p(h-L)] + \frac{2\lambda_p^2}{\lambda_p^2 + \lambda_G^{*2}} - \frac{2\lambda_p}{\lambda_p^2 + \lambda_G^{*2}} \exp[-\lambda_G^*(h-L)] \\ &\quad \left. * (\lambda_G^* \sin[\lambda_p(h-L)] + \lambda_p \cos[\lambda_p(h-L)]) \right\} \end{aligned} \quad (13)$$

Finally, the planar (single component) gust angle of attack standard deviation is just

$$\sigma_\alpha = \frac{\sqrt{\text{var}(V_z(h))}}{U} \quad (14)$$

The consequences of Eq. (14) can easily be explored numerically. For example, take $\lambda_G^* = 0.00565$ rad/m ($l_G = 300$ m), the planar gust velocity standard deviation $\sigma_G = 1$ m/s, a typical pitch wave length = 244 m ($\lambda_p = 0.02577$ rad/m), the launcher length $L = 4.57$ m, and the axial acceleration $a = 4.66$ g. These numbers are typical for small sounding rockets [3]. Next, plot the angle of attack as a function of altitude as shown in Fig. 4. Above altitudes of about two pitch/yaw wavelengths, the angle of attack is seen to approach an asymptotic value. This asymptote can be easily found to be

$$\lim_{h \rightarrow \infty} \sigma_\alpha = \sqrt{\frac{\lambda_p^2 \lambda_G^* \sigma_G^2}{2a(\lambda_p^2 + \lambda_G^{*2})}} \quad (15)$$

Thus, Fig. 4 shows that the typical asymptotic planar standard deviation in gust angle of attack is about 0.008 rad. $\approx \frac{1}{2}$ deg.

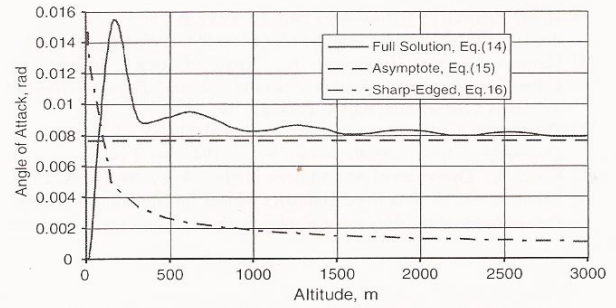


Fig. 4 Standard deviation in planar gust angle of attack.

Numerical studies [3] have shown this result to be insensitive to the pitch wave number.

Lastly, compare these results with the classical sharp-edged (step function) gust response. Ignoring any initial overshoot, this becomes

$$\sigma_\alpha = \frac{\sigma_G}{\sqrt{2ah}} \quad (16)$$

The sharp-edged gust response is shown in Fig. 4.

Discussion

Several remarks are now appropriate. The sounding rocket boost-phase gust angle of attack model presented in this paper relies on several key assumptions. The most important include neglecting short-period damping and the use of an approximate isotropic turbulence model for flight in the planetary boundary layer. Errors due to these two can be roughly estimated to be $\sim 15\%$ each. Probably the most important source of error arises from the fact that many sounding rockets are launched from remote regions on poorly controlled schedules making a priori collection of good geophysical data nearly impossible, even apart from any challenges in the measurement of gust data. More sophisticated gust response models could doubtless be developed, but, because of the uncertainties in the gust data used, they should not be expected to provide significant improvement in accuracy.

Conclusions

A simple physics-based estimate of sounding rocket boost-phase gust angle of attack statistics has been developed and compared with the older step-function gust model. First, this paper shows that the standard deviation in planar angle of attack has a transient peak at low altitude but quickly approaches its constant asymptotic value. The asymptote provides a simple engineering result suitable for many practical problems.

Finally, the results show that the sharp-edged step-function gust model, once commonly used in blind ignorance, is a poor approximation. The sharp-edged gust model has no mechanism to accumulate or dissipate short-period gust response energy, an essential physical feature of the problem. Because short-period damping has been neglected, the model described in this paper allows gust response energy in the short-period mode to accumulate without bound. Because sounding rocket powered flight is often brief, there will usually not be enough time/altitude for damping to remove much energy. Thus, using the current model will provide conservative results, especially at higher altitudes.

It is recommended that the results developed here be used in the future to estimate sounding rocket structural loads during boost phase.

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